where

$$Z_{me}^{xx}(m,n) = \frac{-1}{4\pi NM} \sum_{k=1}^{N} \sum_{L=1}^{M} I_{m,n,k,L}^{xx} \tag{31}$$

$$I_{m,n,k,L}^{xx} = \int_{l_{n,k}^{e}} \int_{l_{m,L}^{m}} V(x_L, y, x', y_k)\, dy dx' \tag{32}$$

where N and M represent the number of Gaussian quadrature nodes on the magnetic and electric segments, respectively. x_L and y_k are the Gaussian quadrature nodes on the aperture and microstrip, respectively, and $l_{n,k}^{e}$ and $l_{m,L}^{m}$ are the lengths of the electric and magnetic segments, respectively. Also

$$V(x,y,x',y') = 2h \frac{(-jk_1 r_0 - 1)}{r_0^3} e^{-jk_1 r_0} \tag{33}$$

where

$$r_0 = \sqrt{(x-x')^2 + (y-y')^2 + h^2}. \tag{34}$$

On the other hand

$$Z_{me}^{zx}(m,n) = \frac{-1}{4\pi NM} \sum_{i=1}^{N_{zx}} a_i^{zx} \sum_{k=1}^{N} \sum_{L=1}^{M} I_{m,n,i,k,L}^{zx} \tag{35}$$

where N_{zx} and a_i^{zx} are, respectively, the number and amplitude of the complex images of $G_{A_{zx}}$. Thus

$$I_{m,n,i,k,L}^{zx} = \int_{l_{i,n,k}^{e}} \int_{l_{m,L}^{m}} V'(x_L, y, x', y_k)\, dy\, dx' \tag{36}$$

$V'(x,y,x',y')$ is shown in (37) at the bottom of the previous page, and

$$r_i = \sqrt{\rho^2 + \left(-jb_i^{zx}\right)^2}. \tag{38}$$

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REFERENCES

[1] W. Grabherr and W. Menzel, "A new transition from microstrip line to rectangular waveguide," in *Proc. of 22nd EuMC*, Espoo, Finland, 1992, pp. 1170–1175.
[2] L. Hyvonen and A. Hujanen, "A compact MMIC-compatible microstrip to waveguide transition," in *IEEE MTT-S Dig.*, San Francisco, CA, June 1996, pp. 875–878.
[3] N. Herscovici and D. Pozar, "Full-wave analysis of aperture-coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1108–1114, July 1991.
[4] A. A. Omar and Y. L. Chow, "A solution of coplanar waveguide with air-bridges using complex images," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 2070–2077, Nov. 1992.
[5] Y. L. Chow, J. J. Yang, D. G. Fang, and G. E. Howard, "A closed form spatial Green's functions for thick microstrip substrates," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 588–593, Mar. 1991.
[6] K. Kunz and R. Luebbers, *The Finite Difference Time Domain Method for Electromagnetics*. Boca Raton, FL: CRC Press, 1993.
[7] J. Cheng, N. I. Dib, and P. B. Katehi, "Theoretical modeling of cavity-backed patch antennas using a hybrid technique," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 1003–1013, Sept. 1995.
[8] J. Yook, N. I. Dib, and P. B. Katehi, "Characterization of high frequency interconnects using finite difference time domain and finite element methods," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1727–1736, Sept. 1994.
[9] R. S. Elliott, *Antenna Theory and Design*. Englewood Cliffs, NJ: Prentice-Hall, 1981.
[10] D. G. Fang, J. J. Yang, and K. Sha, "The exact images of a horizontal magnetic dipole above or within a microstrip substrate," in *Proc. Sino-Japanese Joint Meeting Optical Fiber Sci. and EM Theory*, Nanjing, China, 1987, pp. 693–698.
[11] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1991, pp. 23–26.
[12] A. W. Glisson and D. R. Wilton, "Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 593–603, Sept. 1980.
[13] G. E. Howard, "Analysis of passive and active microwave integrated circuits by the field approach," M.A.Sc. thesis, Univ. Waterloo, Waterloo, Ont., Canada, 1988.
[14] R. F. Harrington, *Field Computation by Moment Methods*. Malabar, FL: R. E. Krieger, 1968, pp. 62–81.
[15] D. A. Huber, "A moment method analysis of stripline circuits through multi-pipe field modeling," M.A.Sc. thesis, Univ. Waterloo, Waterloo, Ont., Canada, 1991.
[16] R. W. Hamming, *Numerical Methods for Scientists and Engineers*. New York: Dover, 1973, pp. 620–622.
[17] A. A. Omar and Y. L. Chow, "Coplanar waveguide with top and bottom shields in place of air-bridges," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1559–1563, Sept. 1993.

Effect of Conductor Backing on the Line-to-Line Coupling Between Parallel Coplanar Lines

Kwok-Keung M. Cheng

***Abstract*— A good estimate of the coupling effect between parallel coplanar waveguide (CPW) lines is important, especially for monolithic microwave integrated circuit (MMIC) applications where unnecessary crosstalk between conductors could be a serious problem. This paper shows how these coupling parameters may be analytically obtained in the presence of the back-face metallization. Closed-form formulas are developed for evaluating the quasi-TEM characteristic parameters based upon the conformal-mapping method (CMM). Very good agreement is observed between the values produced by these formulas and by a spectral-domain method (SDM).**

***Index Terms*—Coplanar waveguide, coupled lines.**

I. INTRODUCTION

Coplanar waveguide is often considered to have free space above and below the dielectric substrate. However, this configuration has been found unsuitable for monolithic microwave integrated circuits (MMIC's), where the substrate is typically thin and fragile. Practical realizations of coplanar waveguides (CPW's) usually have an additional ground plane beneath the substrate. The main advantages of this back-face metallization are principally to increase the mechanical strength as well as to improve heat dissipation. The standard CPW, plus this additional conducting ground plane, is often called conductor-backed CPW (CBCPW). Various approaches have been reported on the characterization of coplanar transmission lines such as the finite-difference method [1], the spectral-domain method

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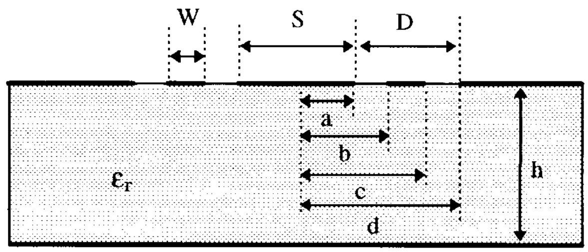

Fig. 1. Coplanar lines separated by a finite ground plane.

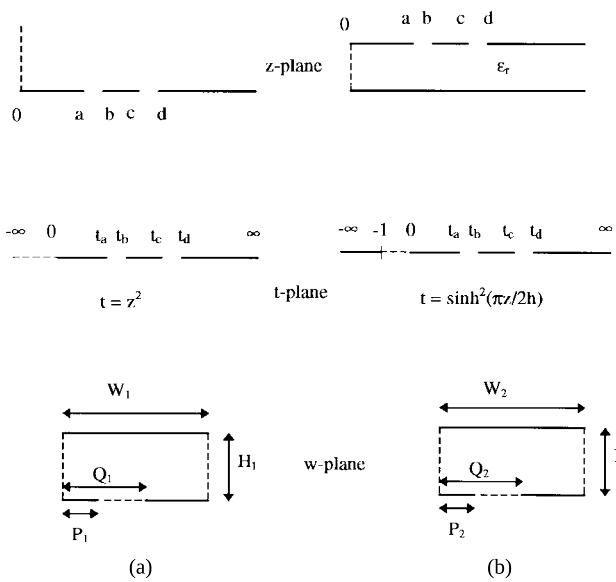

Fig. 2. Even-mode capacitance evaluation by conformal transformations. (a) Air region. (b) Dielectric layer.

(SDM) [2], [3], and the conformal-mapping method (CMM) [4]. Ideally, the spacing between CPW's should be large in order to avoid line-to-line coupling in circuits. However, in practice, the ground-plane width (Fig. 1) should be as small as possible, since it has a direct influence upon the actual circuit size. A compromise between these two constraints can be suggested by a quantitative estimate of the coupling coefficient. Previous analyses [4], [5] of the coupling effects between CPW pairs were primarily based on the assumption of infinite thick substrate. In this paper, the effect of the conductor backing on the line-to-line coupling between CPW's with a finite ground-plane separation is examined. Closed-form expressions, accounting for the aforementioned effect, is derived based upon the CMM. Numerical results produced by the SDM are also included for comparison.

II. Analysis

The structure to be analyzed is shown in Fig. 1, where two CPW's are separated by a ground plane of width $2a$. All conductors are assumed to be infinitely thin and perfectly conducting. It is assumed that the air-dielectric interfaces, where all the conductors are located, can be dealt with as though perfect magnetic walls are present in them. The even- and odd-mode capacitances per-unit-length of the structure can thus be considered as the sum of the capacitances in the air region and in the dielectric layer. The odd-mode capacitance is evaluated by assuming an electric wall is present at the center of the structure. Hence, the capacitance per-unit-length for the odd mode can be obtained through a sequence of conformal mappings [4] as

$$C_o(\varepsilon_r) = \varepsilon_0 \frac{K(k_{o1})}{K'(k_{o1})} + \varepsilon_0 \varepsilon_r \frac{K(k_{o2})}{K'(k_{o2})} \tag{1}$$

$$k_{o1} = \sqrt{\frac{(d^2-a^2)(c^2-b^2)}{(c^2-a^2)(d^2-b^2)}} \tag{2a}$$

$$k_{o2} = \sqrt{\frac{[\sinh^2(\zeta d) - \sinh^2(\zeta a)][\sinh^2(\zeta c) - \sinh^2(\zeta b)]}{[\sinh^2(\zeta c) - \sinh^2(\zeta a)][\sinh^2(\zeta d) - \sinh^2(\zeta b)]}} \tag{2b}$$

where $\zeta = \pi/2h$, $K(k)$, and $K'(k)$ are the complete elliptic integral of the first kind and its complement. Similarly, the even mode is analyzed by placing a magnetic wall at the center of the structure. The even-mode capacitance per-unit-length is evaluated through a sequence of transformations as depicted in Fig. 2. The upper region on the t-domain is mapped onto the w-domain through the mapping

$$w = \int_{t_a}^{t} \frac{dt}{\sqrt{(t-t_a)(t-t_b)(t-t_c)(t-t_d)}} \tag{3}$$

and by the following equations:

$$\frac{W_1}{H_1} = \frac{K(k_{o1})}{K'(k_{o1})} \tag{4a}$$

$$\frac{P_1}{W_1} = \frac{F\left(\arcsin\sqrt{\frac{a^2(d^2-b^2)}{b^2(d^2-a^2)}}, k_{o1}\right)}{K(k_{o1})} \tag{4b}$$

$$\frac{Q_1}{W_1} = \frac{F\left(\arcsin\sqrt{\frac{d^2-b^2}{d^2-a^2}}, k_{o1}\right)}{K(k_{o1})} \tag{4c}$$

$$\frac{W_2}{H_2} = \frac{K(k_{o2})}{K'(k_{o2})} \tag{5a}$$

$$\frac{P_2}{W_2} = \frac{F\left(\arcsin\sqrt{\frac{\sinh^2(\zeta a)[\sinh^2(\zeta d)-\sinh^2(\zeta b)]}{\sinh^2(\zeta b)[\sinh^2(\zeta d)-\sinh^2(\zeta a)]}}, k_{o2}\right)}{K(k_{o2})} \tag{5b}$$

$$\frac{Q_2}{W_2} = \frac{F\left(\arcsin\sqrt{\frac{\cosh^2(\zeta a)[\sinh^2(\zeta d)-\sinh^2(\zeta b)]}{\cosh^2(\zeta b)[\sinh^2(\zeta d)-\sinh^2(\zeta a)]}}, k_{o2}\right)}{K(k_{o2})} \tag{5c}$$

where $F(\phi, k)$ is the incomplete elliptic integral of the first kind, written in Jacobi's notation. Subsequently, the capacitance of the rectangular structure in the w-domain can be considered as the sum of two capacitances [6] and the total capacitance per-unit-length for the even mode is, therefore, given by

$$C_e(\varepsilon_r) = \varepsilon_0 C_p(W_1/H_1, P_1/W_1, Q_1/W_1) + \varepsilon_0 \varepsilon_r C_p(W_2/H_2, P_2/W_2, Q_2/W_2) \tag{6}$$

where

$$C_p(\alpha, \beta, \gamma) = K(k_1)/K'(k_1) + K(k_3)/K'(k_3) \tag{7}$$

$$\frac{F[\arcsin(k_1/k_2), k_2]}{K(k_2)} = \frac{\beta}{\delta} \tag{8a}$$

$$\frac{F[\arcsin(k_3/k_4), k_4]}{K(k_4)} = \frac{1-\gamma}{1-\delta} \tag{8b}$$

$$\frac{K(k_2)}{K'(k_2)} = \alpha\delta \tag{8c}$$

$$\frac{K(k_4)}{K'(k_4)} = \alpha(1-\delta) \tag{8d}$$

$$\delta = (\beta+\gamma)/2 \tag{9}$$

Simple and accurate formulas are available [7] for solving (4), (5), and (8). Hence, the odd- and even-mode characteristic impedances

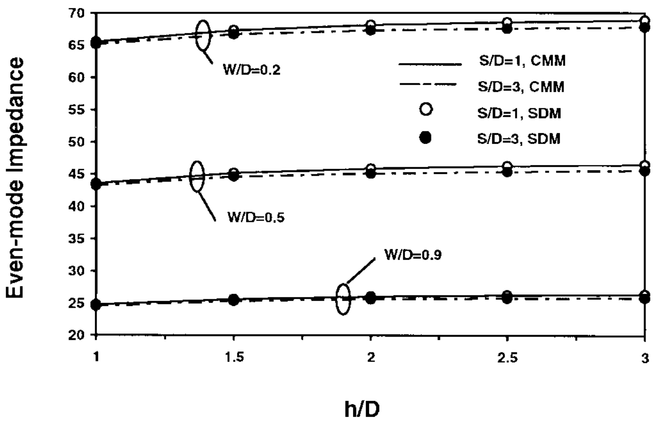

Fig. 3. Even-mode impedance values versus h/D and S/D.

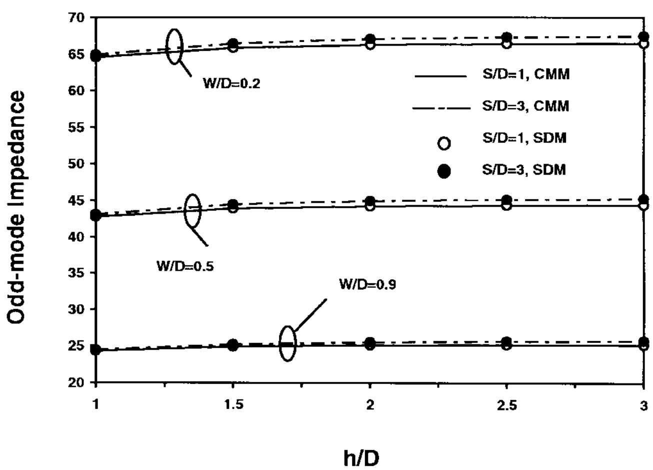

Fig. 4. Odd-mode impedance values versus h/D and S/D.

and coupling coefficient of the coupled CPW's may be evaluated by the well-known expressions

$$Z_{o(o,e)} = c_v^{-1}[C_{o,e}(\varepsilon_r)C_{o,e}(1)]^{-1/2} \tag{10}$$

$$C = -20\log\left(\frac{Z_{o,e} - Z_{o,o}}{Z_{o,e} + Z_{o,o}}\right) \tag{11}$$

where c_v is the velocity of light in vacuum.

III. Numerical Results and Discussions

Figs. 3–5 shows the even- and odd-mode characteristic impedance and coupling coefficient values of coupled CPW's $(\varepsilon_r = 12.9)$, evaluated by the proposed formulas, for different aspect ratios ($W/D = 0.2,\ 0.5,\ 0.9$). In these examples, the signal conductors are placed midway between the upper ground planes (i.e., $b - a = d - c$). For purposes of comparison, the result for the same structure calculated by the SDM [8] are also included in Figs. 3–5. For the SDM used here, the potential functions in the slots are divided into N subsections. N is chosen by increasing the number of subsections until the resulting impedance value does not vary by more than 0.1%. Note that the discrepancies between the values obtained by the two approaches are small (less than 1% error in impedance levels and 0.3 dB in coupling coefficient calculated). It can also be observed that the even- and odd-mode characteristic impedances increase gradually with increasing value of h/D. However, the coupling coefficient is a strong function of both the ratios S/D and h/D, as expected. For instance the coefficient drops by almost 7 dB ($S/D = 1$) when the ratio h/D increases from 1 to 3. Furthermore, the coupling between CPW's is higher for low impedance lines (as W/D approaches 1).

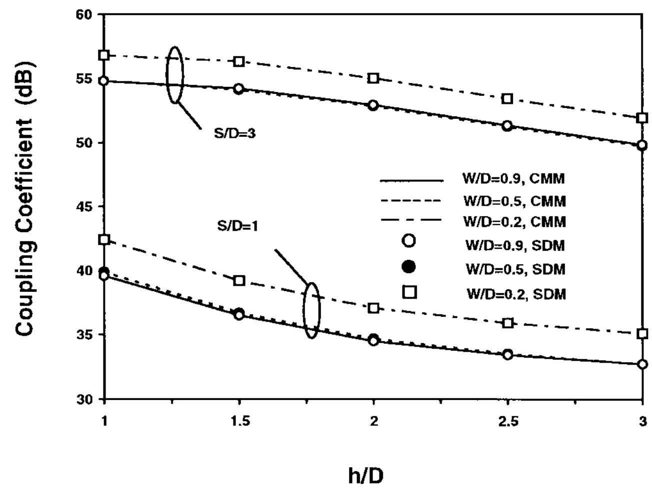

Fig. 5. Coupling coefficient versus h/D and S/D.

IV. Conclusion

It has been shown that the presence of the back-face metallization is an influential factor on estimation of the coupling between parallel CPW's, especially for MMIC applications where unnecessary crosstalk between conductors could be a serious problem. A closed-form analytical solution has been devised for obtaining the quasi-TEM parameters of coupled CPW lines. Numerical results generated by the proposed method is in excellent agreement with those values obtained by an SDM. These formulas are both accurate and easy to implement, thus making it an excellent choice for use in computer-aided design (CAD)-oriented tools.

References

[1] T. Hatsuda, "Computation of coplanar-type strip-line characteristics by relaxation method and its application to microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 795–802, Oct. 1975.

[2] Y. C. Shih and T. Itoh, "Analysis of conductor-backed coplanar waveguide," *Electron. Lett.*, vol. 18, no. 12, pp. 538–539, June 1982.

[3] R. R. Boix and M. Horno, "Modal quasi-static parameters for coplanar multiconductor structures in multilayered substrates with arbitrary transverse dielectric anisotropy," *Proc. Inst. Elect. Eng.*, pt. H, vol. 136, no. 1, pp. 76–79, Feb. 1989.

[4] G. Ghione and C. Naldi, "Coplanar waveguides for MMIC applications: Effect of upper shielding, conductor backing, finite-extent ground planes, and line-to-line coupling," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 260–267, Mar. 1987.

[5] N. H. Zhu, W. Qiu, E. Y. B. Pun, and P. S. Chung, "Analysis of a coupled coplanar waveguide for MMIC applications," *Microwave Opt. Technol. Lett.*, vol. 10, no. 3, pp. 182–186, Oct. 1995.

[6] K. K. M. Cheng, "Analysis and synthesis of coplanar coupled lines on substrates of finite thickness," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 636–639, Apr. 1996.

[7] K. K. M. Cheng and I. D. Robertson, "Quasi-TEM study of microshield lines with practical cavity sidewall profiles," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2689–2694, Dec. 1995.

[8] K. K. M. Cheng and J. K. A. Everard, "A new technique for the quasi-TEM analysis of conductor-backed coplanar waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1589–1592, Sept. 1993.